

Cosmology Glossary

Distance

Astrophysical distances are measured in parsecs (3.12 light years), defined as the distance subtended by an angle of 1 arcsecond with a baseline of 1 AU (astronomical unit, the earth's orbital radius).

$$1 \text{ pc} = 3.12 \times 10^{18} \text{ cm}$$

Cosmological Distance:

"True" distance to astrophysical objects depends on

1. Relativistic Redshift: z
2. Hubble Constant: H_0
3. Mass Density: Ω_M
4. Energy Density: Ω_{DE}
5. Curvature of Spacetime: k

Magnitudes

Observed brightness B is $B = L / 4\pi D^2$

where L is the intrinsic brightness and D is the distance to the source

Magnitude is the unit of apparent brightness

$$m = -2.5 \log B$$

$$= -2.5 \log L / 4\pi D^2$$

The absolute magnitude of a source is the apparent magnitude it would have at a distance of 10pc:

$$M = -2.5 \log L + 5 + \text{constant}$$

Cosmological Redshift :

This is basically the Doppler effect, where the frequency or wavelength shift of the emitted radiation is proportional to the relative velocity between the source and the detector -- i.e. approaching or receding. Cosmologically, the relative velocity is due to the expansion of space-time, not to the motion of the sources. Mathematically the redshift z is defined:

$$z = (\lambda_{\text{obs}} - \lambda_{\text{em}}) / \lambda_{\text{em}} = v_{\text{rec}} / c$$

$$z = \lambda_{\text{obs}} / \lambda_{\text{em}} - 1$$

Relativistic Redshift

At large values of recession velocity, i.e. when $z > 0.1$, have to take into account relativistic effects. Then the redshift becomes:

$$z = \sqrt{\frac{c + v}{c - v}} - 1$$

$$\frac{v_{\text{true}}}{c} = \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1}$$

Hubble Constant: H_0

$$v = H_0 D$$

H_0 is the expansion rate of the universe at a given time. D is the distance to a source, and v is the velocity of recession. The current value of the expansion parameter is

$$H_0 = 67 \text{ km/s/Mpc}$$

Cosmological Constant: Λ

Einstein's "fudge factor", considered to be energy of the vacuum. It has negative pressure, and its equation of state parameter $w = -1$ (see definition below).

Critical Density: Δ_c

when curvature $k=0$, and the cosmological constant $\Lambda=0$, then the critical density is

$$\Delta_c = 3H^2 / 8\pi G$$

Mass Density ratio: Σ_M

Σ_M is the ratio of the measured mass density to the critical density:

$$\Sigma_M = \Delta_M / \Delta_c$$

The total matter density is sum of baryonic and dark matter $\Sigma_M = \Sigma_B + \Sigma_{DM}$. Matter density decreases with time as the universe expands.

Energy - Mass Density ratio: Σ_T

The total energy-mass density ratio is the sum of mass and energy density:

$$\Sigma_T = \Sigma_M + \Sigma_{DE}$$

where $\Sigma_{DE} = 3\Lambda / 8\pi H^2$, (vacuum) energy density. A flat universe, $k=0$, has $\Sigma_T=1$

Equation of State of the Universe

$$p = w\rho \rightarrow \rho \propto R^{-3(1+w)}$$

where w can vary with time (or equivalently with redshift z). The equation of state is the relationship between pressure p and density Δ of stuff (matter and/or energy).